Economies of scope or specialisation in Polish dairy farms –
an application of a new local measure

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Abstract
Traditional measures of economies of scope, based on the definition introduced by Baumol et al. (1982), have serious shortcomings. However, when different products are expressed in the same units (e.g., using constant, reference prices), we can define a useful class of indices by considering the 1% increase in aggregate production through either the appropriate increase of only one product or the 1% increase of all products. The ratio of costs in these two situations leads to the coefficient of the effect of production structure change, which is our local counterpart of traditional measures of economies of scope or specialization. It has been illustrated using panel data and Bayesian inference in a two-product translog cost frontier model for Polish dairy farms.

Keywords: multiproduct cost function, stochastic frontier model, Bayesian inference, agricultural economics

JEL Classification: D22, C23, C51, Q12

1 Cost effects of production structure in multiproduct settings

Microeconomic (frontier) cost functions have been widely used in empirical analyses of costs of production. One of important areas is the analysis of economies of scope or specialisation of producers, with theoretical foundations presented by Panzar and Willig (1981). This area was particularly important in the banking sector in 1980s and 1990s, due to interest in measuring economic consequences of mergers and acquisitions; see Kim (1986), Berger et al. (1987), Lawrence (1989), Dietsch (1993), Hughes and Mester (1993), Mester (1993), Muldur and Sassenou (1993), Zardokoohi and Kolari (1994), Marzec and Osiewalski (2001).

Let \( Q = (Q_1, \ldots, Q_G) \in \mathbb{R}_G^+ \) be the vector of quantities of \( G \) products, \( C(Q_1, \ldots, Q_G; x, \delta) \) - the cost function, \( x \) - the vector of factor prices (and the quantities of fixed inputs in the case of short-run analysis) and \( \delta \) - the vector of parameters. Economies of scope are present when

\[
C(Q_1, \ldots, Q_G; \delta) < C(Q_1, 0, \ldots, 0; \delta) + C(0, Q_2, 0, \ldots, 0; \delta) + \ldots + C(0, \ldots, 0, Q_G; \delta).
\]

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that is when the cost function is sub-additive, so that the cost of producing \( Q=(Q_1,\ldots,Q_G) \) in one unit is smaller than the sum of costs of producing each product in a separate, specialised unit. If the inequality goes in the opposite direction, there are positive effects of specialisation.

Baumol et al. (1982) proved that, in the case of any twice differentiable multiproduct cost function, a sufficient (but not necessary) condition for economies of scope is

\[
\frac{\partial^2 C(Q;x,\delta)}{\partial Q_g \partial Q_j} < 0 \quad \text{for } g \neq j, \quad g, j = 1,\ldots,G.
\] (2)

The inequality in (2) means that the marginal cost of each individual product is a decreasing function of any other product. That is, slightly increasing any other product \((Q_1,\ldots,Q_{g-1},Q_{g+1},\ldots,Q_G)\) results in a smaller value of the marginal cost of product \(g\).

Baumol et al. (1982) proposed the coefficient that follows directly from the definition in (1):

\[
SC = \frac{\sum_{g=1}^{G} C_g(Q_g;x,\delta) - C(Q;x,\delta)}{C(Q;x,\delta)},
\] (3)

where \( C_g(Q_g;x,\delta) = C(0,\ldots,0,Q_g,0,\ldots,0;\cdot) \) is the cost of producing \(Q_g\) and no other products; \(|SC|\cdot100\%\) represents the percentage cost reduction (if \(SC>0\)) or increase (if \(SC<0\)) due to joint production (as compared to producing each product separately, in a specialised unit). This coefficient was extended to a group of products, see Kim (1986).

However, the definition of economies of scope and the corresponding coefficient \(SC\) can be impractical in empirical studies as \(C_g(Q_g;x,\cdot)\) is based on zero levels of all other products, i.e. on values usually not met in the data. This may mean extrapolating the cost function far outside the region of reasonable approximation. Moreover, the popular Cobb-Douglas and translog forms of the cost function are defined only for strictly positive arguments. These considerations led to more practical economies of scope coefficients. In the case of only two products Zardkoohi and Kolari (1994) proposed:

\[
SCP = \frac{\Delta C_1 + \Delta C_2 - \Delta C_{1,2}}{\Delta C_{1,2}}, \quad \text{where:}
\]

\[
\Delta C_1 = C(Q_1^{\min} + \Delta Q_1, Q_2^{\min};x,\delta) - C(Q_1^{\min}, Q_2^{\min};x,\delta)
\]

\[
\Delta C_2 = C(Q_1^{\min}, Q_2^{\min} + \Delta Q_2;x,\delta) - C(Q_1^{\min}, Q_2^{\min};x,\delta)
\]

\[
\Delta C_{1,2} = C(Q_1^{\min} + \Delta Q_1, Q_2^{\min} + \Delta Q_2;x,\delta) - C(Q_1^{\min}, Q_2^{\min};x,\delta);
\]

\(Q_1^{\min}\) and \(Q_2^{\min}\) are the smallest production levels observed in the data, \(\Delta Q_1\) and \(\Delta Q_2\) denote the differences between the smallest and average production levels, \(\Delta C_{1,2}\) is the difference in
the cost of producing both products on their average and minimum levels, and $\Delta C_1$ is an additional cost of producing $\Delta Q_1$ extra units above the minimum level of the first product (keeping the minimum level of the second product). While $SCP$ is well defined for any functional form of the cost function, it is based on the increase of production from the minimum to the average level in the dataset. This may mean changes that are too large and do not represent production possibilities of any real production unit. Other, slightly modified, indices of the so-called **within-sample economies of scope** were proposed by Mester (1993) and Hughes and Mester (1993).

In order to avoid problems with using too large increases in production, Marzec and Osiewalski (2001) proposed a truly local measure of economies of scope or specialisation. The crucial assumption is that we can express the levels of all products in the same units. In practice, this will often mean that we have to resort to monetary units by using some constant prices of $G$ products. Then the aggregate production $Q^A = Q_1 + \ldots + Q_G$ can be calculated and the proposed coefficient of the **effect of production structure change (EPSC)** is:

$$EPSC_g(r) = \frac{C(Q_1, \ldots, Q_g + r \cdot Q^A, \ldots, Q_G; x, \delta)}{C(Q_1 \cdot (1+r), \ldots, Q_g \cdot (1+r), \ldots, Q_G \cdot (1+r); x, \delta)}$$  for $r \in (-u_g, +\infty)$,  

where $u_g = Q_g/Q^A$ and $r$ denotes the assumed change in the scale of production. $EPSC_g(r) < 1$ means that the cost of increasing the level of product $g$ (only) by $r \cdot Q^A$ units is smaller than the cost of simultaneously increasing each product by $r \cdot Q_h$ units ($h=1,\ldots,G$). Since the latter corresponds to increasing scale of production without changing its structure, $EPSC_g(r) < 1$ indicates that the change of the share of product $g$ in the aggregate product from $u_g$ to $(u_g+r)/(1+r)$ leads to the cost reduction by $(1-EPSC_g)\times100\%$.

2 Short-run translog frontier cost function for Polish dairy farms

The Bayesian frontier cost model presented in this study is estimated using balanced panel data from 846 Polish dairy farms observed over the period 2004-2011 (8 years); the data come from the Farm Accountancy Data Network (FADN). The construction of the variables is based on other studies (on dairy farms) in which FADN data were used (Maietta, 2000; Frahan et al., 2011).

In the present study we consider a variable cost function with two output categories: production of milk ($Q_1$, including milk products) and other production ($Q_2$, i.e. total crop production, livestock output and livestock products except milk) and five input categories.
Both products are expressed as the deflated total net farm revenues from sales excluding the value of feed, seeds and plants produced on the farm.

Variable inputs consist of the following: capital (buildings and machinery, \( K \)), materials and services (\( M \)), utilised agricultural area (\( A \)), herd of dairy cows (\( Z \)). We assume that labour (\( L \)) is a fixed factor; hence it means that it was not subject to optimization. The cost of capital is measured by the sum of financial expenses – in particular, the costs of repairs and maintenance of capital equipment, interest paid, and annual depreciation on buildings and machinery. The category “materials and services” aggregates mainly expenses on fertilisers, pesticides, seeds, feeds, fuel, energy and veterinary services. The utilised agricultural area is the value of owned and rented land. The herd of dairy cows corresponds to the yearly average value of dairy cows. The price of capital is obtained by dividing the cost of capital by the value of capital. The price of intermediate inputs is constructed by aggregating Laspyares indices of the components weighted by farm-specific cost shares. The resulting series are farm specific due to differences in input composition. The price of area at the farm level is measured by the rental rates for farm land provided by FADN. When a farm uses only own area, the price is calculated as the average rental rate (at the same year) over the farms belonging to the same region. The price of herd was calculated using FADN data; it was obtained by dividing the value of dairy cows by their number. Labour input is measured as the total labour (family and hired) expressed in hours. The variable cost (\( VC \)) is computed as the sum of all costs associated with four variable inputs used in the production process.

In this study the cost frontier is formulated by a short-run cost function which relates the observed variable cost to output quantities, prices of variable inputs and quantities of fixed inputs, allowing for inefficiency and random noise. The stochastic cost frontier model based on panel data is defined as

\[
y_{it} = h_t(x_{it}, \beta) + v_{it} + z_{it}, \quad (i = 1, \ldots, N; t = 1, \ldots, T),
\]

where \( y_{it} \) is the natural logarithm of the observed cost for unit \( i \) at time \( t \) \((i=1, \ldots, N; t=1, \ldots, T)\); \( x_{it} \) is a row vector of exogenous variables; \( h_t \) is a known parametric functional form with \( \beta \) as a \((k \times 1)\) vector of parameters, \( v_{it} \) and \( z_{it} \) are random terms, one symmetric about zero and the other non-negative. In the case of a cost frontier, the inefficiency term \( z_{it} \) captures the overall cost inefficiency, reflecting cost increases due to both technical and allocative inefficiency of farm \( i \) at time \( t \). Here the inefficiency term may reflect not only a farm specific effect but also a time-varying component. Cost efficiency score is calculated as \( r_{it}=\exp(-z_{it}) \), which is an easily interpretable quantity in the interval \((0; 1]\).
Empirical analyses require a particular functional form of the cost function. In (6) we use the translog specification, since it is a local second-order Taylor series approximation of any sufficiently smooth “true” cost function. The estimated translog frontier should be a cost function, so linear homogeneity in input prices is imposed by normalising both the cost and input prices by one input price (e.g., by $w_2$). Consequently, for $y_{it}$ defined as the log cost minus $\ln(w_{Z,it})$, the functional form of the variable cost model is:

$$h_i(x_{it}; \beta) = \beta_0 + \sum_{h=1}^{6} \beta_h x_{it,h} + \sum_{h=1}^{6} \sum_{j=1}^{6} \beta_{hj} x_{it,h} x_{it,j} + \beta_{Q1} \cdot t \cdot \ln Q_{it,1} + \beta_{Q2} \cdot t \cdot \ln Q_{it,2} + \beta_{trend,1} \cdot t + \beta_{trend,2} \cdot t^2,$$

(7)

$$x_{it} = \begin{bmatrix} \ln \frac{W_{K,it}}{w_{Z,it}} & \ln \frac{W_{M,it}}{w_{Z,it}} & \ln \frac{W_{A,it}}{w_{Z,it}} & \ln L_{it} & \ln Q_{1,it} & \ln Q_{2,it} \end{bmatrix},$$

where $w_h$ is the price of the variable input $h$ ($h \in \{K, M, A, Z\}$). Additionally, the trend variable $t$ has been introduced in order to capture the influence of technical progress; it allows to model the variability of returns to scale (RTS) over the time period considered.

To define our statistical model, we make the usual assumption that the symmetric error terms $\nu_{it}$ are independent and normally distributed with the same unknown variance, i.e., they are $iid \ N(0, \sigma^2)$. This study employs the standard Bayesian normal-exponential stochastic frontier model with Varying Efficiency Distribution (VED) specification proposed by Koop et al. (1997). Subsequently, the inefficiency terms vary over time and production units, but not freely. Namely, $z_{it}$ are independent and follow the exponential distributions with means (and standard deviations) $\lambda_{it}$ that depend on exogenous variables $s_{it,j}$ ($j=1,...,m$), i.e.

$$\ln \lambda_{it} = -\sum_{j=1}^{m} s_{it,j} \cdot \ln \phi_j$$

where $\phi > 0$ are additional unknown parameters; see also Osiewalski and Steel (1998). The important special case when $m = 1$ and $s_{it,1} = 1$ is called the Common Efficiency Distribution (CED) specification.

In our VED specification we use nine dummy variables to describe $\lambda_{it}$; they explain possible systematic differences in efficiency levels due to some farm characteristics. The binary exogenous variables are: farm size measured by land area (small, large), the economic size and the amount of dairy cows; type of specialization (one when milk production is the main source of farm income, zero otherwise), information on whether the farmer has received less favoured areas subsidies or investment subsidies. Other two factors that could potentially influence farm efficiency are whether the farmer rents land or uses hired labour.
The statistical modelling and inference is based on the Bayesian Stochastic Frontier Analysis, proposed by van den Broeck et al. (1994) and Koop et al. (1997), which is now regarded as being relatively standard. The complexity of the stochastic frontier model requires advanced numerical methods to describe the posterior distribution. As Koop et al. (1997) and Osiewalski and Steel (1998) showed, Gibbs sampling, a relatively simple Markov Chain Monte Carlo algorithm, is an efficient tool for generating samples from the posterior distribution.

3 Results on cost frontier and cost efficiency

In Table 1 we present the posterior means and standard deviations for main characteristics of the cost function. The individual posterior means of cost elasticities with respect to all factor prices and two outputs have the expected sign for almost every farm and every period; this holds particularly strongly for the outputs and the prices of capital, materials and livestock.

Positive posterior means of the cost elasticity with respect to the fixed factor (labour) suggest that farms are far from long-run cost minimisation; about 66% of the estimated elasticities are (slightly) positive. A typical Polish dairy farm is characterised by increasing short-run returns to scale; the RTS coefficient is approximately 1.2. Only 6% of farms operate at decreasing short-run returns to scale and the vast majority of them are the largest producers in the sample.

\[
\begin{array}{cccccc}
\text{Cost elasticity w.r.t.:} & \text{Posterior Mean} & \text{Posterior Std. dev.} & \text{Prior Mean} & \text{Prior Std. dev.} & \text{Positive sign} \\
\text{price of capital} & 0.141 & 0.009 & 0.25 & 0.5 & 98\% \\
\text{price of materials} & 0.617 & 0.016 & 0.25 & 0.5 & 100\% \\
\text{price of area} & 0.064 & 0.010 & 0.25 & 0.5 & 84\% \\
\text{price of livestock} & 0.177 & 0.014 & 0.25 & 0.5 & 99\% \\
\text{production of milk (}Q_1\text{)} & 0.644 & 0.005 & 0.5 & 0.5 & 100\% \\
\text{other production (}Q_2\text{)} & 0.211 & 0.004 & 0.5 & 0.5 & 98\% \\
\text{labour (}L\text{)} & 0.026 & 0.013 & -0.1 & 0.5 & 66\% \\
\end{array}
\]

Table 1. Posterior moments for elasticities of cost with respect to outputs, labour and input prices (for a typical farm, with average values of logs of explanatory variables).

Note that, in Table 1, the posterior standard deviations of elasticities are much smaller than the prior standard deviations. This means that our prior distribution, which imposes
microeconomic regularity in a very weak fashion, has not distorted the evidence coming from the likelihood function, based on quite informative data.

Regarding cost efficiency, the average posterior mean of $r_{it}$ is 0.94, while the average posterior standard deviation is 0.05. This implies that Polish dairy farms could have decreased variable cost by about 6% on average. Most of the farms (91%) were relatively efficient, with efficiency above 0.9. The minimum estimate of cost efficiency (among all the observations in our sample) is 0.5. The detailed results are not reported here due to space constraints.

4 Effects of production structure change in Polish dairy farms

The main goal of this study is to obtain results regarding cost effects of (small) changes in the product structure. The estimates of the $EPSC$ coefficient (5) are presented in Figures 1 and 2. For illustrative purposes, this measure was calculated for three firms of different size and production structure. We assume in our interpretations below that the change in the scale of production ($r$) belongs to the interval $(0; 0.1]$.

The large farm with decreasing returns to scale (Figure 1) is characterized by positive cost effect of the change in production structure if it increases the share of $Q_1$ (milk), although it is already high. The down sloping curve in Figure 1 shows for $r = 0.02$ and $EPSC_1 = 0.9988$ that the change of the share of milk in the aggregate product from 89.1% to 89.2% leads to the cost reduction by 0.12% in comparison to the increase of production scale without changing its structure. The upward sloping curve shows that the cost of increasing the level of $Q_2$ (other production) by 43000 PLN ($0.02 \times 2144000$) is by 0.82% ($EPSC_2 = 1.0082$) greater than the cost of increasing the aggregate product by the same amount without changing the shares of $Q_1$ and $Q_2$. Alternatively, in the case of diminishing total production, reducing only $Q_2$ is advisable because it results in a smaller cost than decreasing scale of production without changing its structure.

The average size farm (Figure 2) has approximately 41% share of $Q_1$ in aggregate product and operates under increasing returns to scale. This medium farm should prefer to increase the share of $Q_2$ in the aggregate product, because it leads to the cost reduction as compared to the situation when this unit expands the scale of production without changing its structure. An increase of the share of $Q_1$ is not desirable; a rise in milk production corresponding to $r = 0.01$ leads to the cost higher by approximately 0.04% ($EPSC_1 = 1.004$) than in the case of proportional growth of $Q_1$ and $Q_2$. Finally, for the small farm in Figure 3 we observe positive cost effects of specialisation in each product, so there are two ways to reduce cost.
This study has illustrated that $EPSC$, defined in (5), is a precise and useful measure of cost effects of changes in production structure.

**Fig. 1.** $EPSC$ for a large farm.

**Fig. 2.** $EPSC$ for a medium farm.

**Fig. 3.** $EPSC$ for a small farm.

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