Uncertainty of the dependence structure and risk diversification effect in Solvency II

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Abstract

This paper is an attempt at estimating the "actual level" of the diversification effect in the process of determining Solvency Capital Requirements (SCR) in Solvency II. At first, the method of determining SCRs in Solvency II is briefly characterised and the role of dependences for the correct specification of diversification effect is presented. This is followed by an analysis of the sensitivity of diversification effect to the dependence structure based on the example of life and health underwriting risk. Cases of the lack of knowledge on the structure, of partial knowledge (of a correlation coefficient only) and of total knowledge are considered (it was assumed that variables are independent and comonotonic).

Keywords: *Solvency Capital Requirements, risk aggregation, VaR bounds, diversification effect*

JEL Classification: C150, C580, G220

1 Introduction

The estimation of the diversification effect on the "actual level" is depended by the proper modelling of dependences among risk factors. Misidentified dependence structure leads to the estimation of the incorrect level of diversification effect, which may cause overestimation or underestimation of capital requirements and may have a considerable impact on the functioning of an insurer and its solvency. In the standard model proposed in the Solvency II system, the variance-covariance method is used to aggregate capital requirements. In this method, dependence is modelled only with the use of linear correlation coefficients. The influence of such a solution on the level of solvency capital requirement of insurers in European Union states was assessed in the fifth Quantitative Impact Study (QIS5). It indicates (see: (*EIOPA Report*..., 2017)) that the diversification effect obtained as a result of applying the aggregation method considerably influences the reduction of solvency capital requirements. In total, such requirements for solo insurers and groups of insurance companies participating in the study were lower by 35.1% (EUR 466 billion).

From the methodological point of view, the variance-covariance method is correct when capital requirements are determined for risk factors subject to multivariate normal (elliptical)

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distribution. When analysing the risk of insurer, this assumption is rarely met (and the creators of the proposed solutions are aware of this). It means that in the analysed standard Solvency II model, the diversification effect is estimated using dependence structures that can describe relations between risk factors in an incorrect way. An obvious question arises: To what extent is the diversification effect estimated in this way reliable?

At first, the method of determining solvency capital requirements used in Solvency II is briefly characterised in the paper and the role of dependences for the correct specification of diversification effect is presented. This is followed by an analysis of the sensitivity of diversification effect to the dependence structure based on the example of life and health underwriting risk. The diversification ratio was estimated applying the standard Solvency II approach, i.e. using the variance-covariance method with the life and health underwriting risk correlation coefficient of 0.5. With a copula, a number of examples were provided to demonstrate that the same correlation coefficient value may correspond to various dependence structures, thus various diversification ratios. The value of the diversification ratio was calculated for the case when the considered risks are independent and comonotonic, and the ratio's minimum and maximum value was calculated assuming no information about the dependence structure between these risks.

2 Risk aggregation and diversification effect in Solvency II

In Solvency II, the principal role in the process of evaluating the solvency of an insurer is played by a solvency capital requirement (SCR). This capital is considered as a cushion against significant deviations from expected loss, whereas coverage for expected loss is provided through provisions. Therefore, it is defined as economic capital, which should guarantee security for the insured if unpredictable losses occur. It is calculated at least once a year and when a considerable change occurred in the risk profile of an insurer. It is assumed that SCR should guarantee with a 0.995 probability that the insurer will be able to meet its obligations within 12 months. It must provide for all measurable risk kinds to which the insurer is exposed.

In the standard Solvency II approach, overall Solvency Capital Requirement for the insurer is calculated with the use of the following formula:

$$SCR = BSCR + Adj + SCR_{Op}\,, \tag{1}$$

where: *BSCR*- Basic Solvency Capital Requirement, *Adj* - adjustment for the risk absorbing effect of technical provisions and deferred taxes, SCR_{Op} - the capital requirement for operational risk.

BSCR value is determined when aggregating SCRs designated for main risk modules (i.e. market risk, counterparty default risk, life underwriting risk, non-life underwriting risk, health underwriting risk, intangible assets risk); SCRs for modules are determined by aggregating SCRs for sub-modules whereas the later result from the aggregation of SCRs for risk drivers[3]. Thus, the process involves 3 aggregation levels which are presented in detail, for example, in (*QIS5 Technical ...*, 2017; Wanat, 2014). It is assumed in the process that not all risks occur simultaneously, so SCR determined for a specific level (e.g. module) is generally not greater than the sum of SCRs set at the -1 level (e.g. for sub-modules). The resulting difference is referred to as the diversification effect (benefit) and it is a key element in the risk management process of an insurer.

If it is formally assumed that on *l*-th ($l = 1,...,3$) aggregation level the capital requirement for *j*-th risk $Y_j^{(l)}$ (insurer[4], module, sub-module) dependent on *k* risks $X_{j1}^{(l-1)},..., X_{jk}^{(l-1)}$ from $l-1$ level (modules, sub-modules, drivers) is determined, the diversification effect can be measured with the use of the diversification ratio (see: e.g. Embrechts et al., 2015):

$$d_j^{(l)} = \frac{\kappa\left(Y_j^{(l)}\right)}{\sum_{i=1}^{k} \kappa\left(X_{ji}^{(l-1)}\right)} \tag{2}$$

where: $\kappa\left(X_{ji}^{(l-1)}\right)$ - capital requirement for risk $X_{ji}^{(l-1)}$, $\kappa\left(Y_j^{(l)}\right)$ - capital requirement for the aggregate risk $Y_j^{(l)}$.

The above formula (2) suggests that the diversification effect depends on the manner of determining capital requirements (henceforth, for the purpose of preserving the transparency of notation, superscripts *l* and subscripts *j* will be omitted) $\kappa(X_i)$ for individual risks X_i and capital requirement $\kappa(Y)$ for aggregated risk *Y*. As already mentioned, these requirements should correspond to economic capital determined for one year, at the confidence level of 0.995. Therefore, in accordance with its definition (cf. e.g. Lelyveld, 2006), $\kappa(X_i)$ and $\kappa(Y)$ should be equal:

$$\kappa(X_i) = VaR_{0.995}(L_i) - \mu_i \tag{3}$$

[3] The manner of determining *Adj* and SCR_{Op} values is presented in (*QIS5 Technical ...*, 2017).
[4] It is obviously BSCR.

$$\kappa(Y) = VaR_{0.995}(L) - \mu \tag{4}$$

where: L_i , μ_i – loss distributions for X_i risks and their expected values, respectively,
L, μ- loss distribution for aggregated risk Y and its expected value, respectively,
$VaR_{0.995}(\cdot)$ - Value-at-Risk at the confidence level of 0.995.

It is, thus, clear that the procedure of estimating capital requirement for aggregated risk Y, which mainly depends on the modelling of dependence structure among variables $L_1,...,L_k$ (so $X_1,...,X_k$ risks) is of key importance to the correct evaluation of the diversification ratio. In the standard Solvency II solution, in case of the aggregation of solvency capital requirements on the second and third level, the variance-covariance method is proposed. The method involves:

- Determining capital requirements for individual risks: $\kappa(X_1),...,\kappa(X_k)$.
- Determining capital requirement $\kappa(Y)$ for aggregated risk Y based on the linear correlation coefficients ρ_{ij} among all pairs of risks $X_1,...,X_k$, in accordance with the following formula:

$$\kappa^{(solv)}(Y) = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{k}\rho_{ij}\kappa(X_i)\kappa(X_j)} . \tag{5}$$

As stated above, $\kappa^{(solv)}(Y)$ should correspond to economic capital necessary for securing potential losses (higher than expected) related to risk Y, in an annual time horizon and at the security level of 0.995. Thus, value $\kappa^{(solv)}(Y)$ obtained as a result of applying the standard procedure (5) should be equal to $\kappa(Y)$ value obtained with the use of formula (4). With the application of formulas (4) and (5), we obtain the same value of solvency capital requirement (i.e. $\kappa^{(solv)}(Y) = \kappa(Y)$) only when (cf. e.g. Dhaene et al., 2005):

i. Capital requirements for individual risks $\kappa(X_1),...,\kappa(X_k)$ are determined in accordance with formula (3).
ii. $L = L_1 + ... + L_k$ and $L_1,...,L_k$ have multivariate normal (elliptical) distribution, which means, in particular, that each variable L_i has normal distribution.

In the process of estimating solvency capital requirements of the insurer, those risks are aggregated which due to their essence are modelled with the use of different distributions and different methods. Therefore, the assumption that they are subject to multivariate normal distribution is ill-founded. A question arises concerning the estimations of the diversification ratio in case we do not know the multivariate distribution of random vector $(L_1,...,L_k)$. An insurer may estimate quite precisely the distribution of losses related to individual risks $X_1,...,X_k$, that is marginals of this vector. Let us attempt to answer the question by assuming that the distributions of variables $L_1,...,L_k$ are known whereas the dependence structure among them is unknown. With this assumption, the diversification effect will only depend on the dependence structure among variables $L_1,...,L_k$ which determines $VaR_{0.995}(L)$. The problem is discussed in more detail in the following section.

3 Value-at-Risk bounds with fixed marginal distributions

Let us assume that on the determined aggregation level, the distributions of random variables $L_1,...,L_k$ have known cumulative distribution functions $F_1,...,F_k$. Then, the distribution of the sum $L = L_1 + ... + L_k$, and the value of $VaR_\alpha(L)$ depend only on the dependence structure of the vector $(L_1,...,L_k)$. Based on Sklar's Theorem, all information on it is in copula *C*. Thus, *k*-th dimensional random vector with fixed marginals $F_1,...,F_k$ and dependence structure in the form of copula *C* will be designated by $(L_1^C,...,L_k^C)$. If the dependence structure between $L_1,...,L_k$ is unknown, we are unable to determine the exact $VaR_\alpha(L)$, and it can only be assumed that it fulfils the following inequalities (see: e.g. Embrechts et al., 2013):

$$\underline{VaR_\alpha}(L) \le VaR_\alpha\left(L_1^C + ... + L_k^C\right) \le \overline{VaR_\alpha}(L) \tag{6}$$

where:

$$\underline{VaR_\alpha}(L) = \inf_{C \in \Xi_k} \left\{VaR_a\left(L_1^C + ... + L_k^C\right)\right\} \tag{7}$$

$$\overline{VaR_\alpha}(L) = \sup_{C \in \Xi_k} \left\{VaR_a\left(L_1^C + ... + L_k^C\right)\right\} \tag{8}$$

whereas Ξ_K denotes the class of all *k*-dimensional copulas.

The issue of seeking bounds (7) and (8) is extremely important from the perspective of risk management and has a long history. The first results in this respect for the sum of two random variables are presented in papers by (Makarov, 1981) and independently in (Rüschendorf, 1982). In recent years, it has been discussed, for example, in (Puccetti and Rüschendorf, 2012a, 2012b, 2014; Embrechts et al. 2013; Puccetti et al., 2013; Bernard et al., 2015, 2016).

The natural outcome of research on respecting *VaR* boundaries was the creation of RA algorithm (*Rearrangement Algorithm*), proposed by Puccetti and Rüschendorf (2012a) and Embrechts et al. (2013). It allows for designating *VaR* boundaries for known but not necessarily identical marginal distributions. In brief, RA algorithm is about constructing dependence functions between random variables L_i by regrouping properly the columns made of random variables so that the distribution of the sum of random variables in convex order would be as low as possible. Research conducted so far indicates that estimation ranges of VaR obtained by AR in various cases are quite broad.

The modification of RA algorithm, also known as ARA (*Adaptive Rearrangement Algorithm*), was proposed by Hofert et al. (2017). The main impulse for the creation of the algorithm was a huge (even if the use of computers is taken into account) number of operations which needed to be performed with the use of RA algorithm in case of a large number of variables L_i. Bernard et al. (2015) constructed ERA algorithm (*Extended Rearrangement Algorithm*) also on the basis of RA algorithm. In relation to RA algorithm, the extension involves determining minimum elements of the distribution of a sum of random variables in the sense of convex order, in the upper and lower part of the distribution separated by the given α with the limitation of variance taken into account. ERA algorithm aims at making the distribution of L as flat as possible on the upper and lower part by applying the RA algorithm on both parts and by moving through the domains in a systematic way in order to satisfy the variance constraint. The examples presented by the authors prove that ERA algorithm works well and an additional condition of limited variance leads to better (as compared to RA algorithm) estimations of *VaR* boundaries. On the basis of ERA algorithm, the authors prove that models used by participants and regulators of the capital market can underestimate *VaR* whereas values-at-risk designated in this manner may be incomparable. They additionally claim that the determination of capital requirements at a high confidence level, e.g. 99.5%, is justified.

4 Diversification effect for life and health underwriting risk – empirical example

This section presents the results of the analysis of the impact of selected dependence structures on the diversification effect in the case of the aggregation of capital requirements for life and health underwriting risk. In the study, it was assumed that losses (in million euros) related to life underwriting risk and health underwriting risk are modelled with the use of random variables of normal distribution[5]: $L_1 \sim N(0, 392)$ and $L_2 \sim N(0, 248)$, respectively. Capital requirements $\kappa(Y)$ and diversification ratio d have been determined:

[5] The parameters adopted were the same as in (Bernard et al. 2016).

- in accordance with the standard procedure proposed in Solvency II, i.e. with the use of formula (5) with correlation coefficient[6] $\rho_{12} = 0.25$;
- with the assumption that the dependence structure between L_1 and L_2 is modelled with the use of the Student copula (df=2), the Student copula (df=5), the Gumbel copula, the Frank copula, the Clayton copula and the Galambos copula. Copula parameters are determined in such a way that the linear correlation coefficient ρ between marginal distributions should be equal to 0.25 in each of the analysed structures. The values of these parameters and additional information on dependences in the lower (λ_L) and upper (λ_U) tail are given in the first column in table 1;
- with the assumption that L_1 and L_2 are independent;
- with the assumption that L_1 and L_2 are comonotonic.

Then, it was assumed that there was no information on the dependence structure between L_1 and L_2, and lower and upper estimations $\kappa(Y)$ and d were determined. $\underline{VaR_{0.995}}(L)$ and $\overline{VaR_{0.995}}(L)$ values necessary for this purpose were obtained with the use of ARA algorithm. The results are given in the second and third column of table 1 and in Fig. 1.

The conducted study indicates that in the process of determining solvency capital requirements in Solvency II, familiarity with only distributions of aggregated risks X_i without familiarity of the dependence structure between them is insufficient. The range of possible values $\kappa(Y)$ from EUR 274.7 to 1791.5 million obtained in this way is useless from a practical perspective. Considering the above, in the process of aggregating capital requirements one should take into consideration dependence between risks. The standard Solvency II solution proposes the use of linear correlation coefficients only. However, as the results of the analysis indicate, the method does not guarantee that the capital will be determined unequivocally. The same correlation coefficients between marginal distributions may correspond to different dependence structures. This results in the estimation of capital $\kappa(Y)$ and the corresponding diversification ratio on different levels. However, it seems natural to expect that additional information on the dependence structure in the form of correlation coefficients between risks should largely narrow down the range of potential values for $\kappa(Y)$ and d. The presumption was confirmed in studies only in the case of several

[6] See: DIRECTIVE 2009/138/EC OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II), ANNEX IV, p. 124.

selected dependence structures (i.e. the Student copula (df=2), the Student copula (df=5), the Gumbel copula, the Frank copula, the Clayton copula and the Galambos copula). Capital values $\kappa(Y)$ from 1234.4 to 1468.1 million were obtained for them, which corresponded to the diversification ratio from the range (74.9, 89.1). Generally, it can be stated that the greater the dependence in the upper tail, the greater the capital requirement $\kappa(Y)$, and thus, the lower the diversification effect. It is just the opposite in case of dependences in the lower tail – the stronger the dependence, the lower the value $\kappa(Y)$ and the greater the diversification effect. The purpose of further research to be undertaken by the authors will be to determine the lower and upper limit for $\kappa(Y)$ and d for any dependence structures.

Dependence structure	**Capital requirement**	**Diversification ratio in %**
Solvency II standard formula	1322.9	80.2
Student copula (df=2, $\rho = 0.265$, $\lambda_L = \lambda_U = 0.278$)	1468.1	89.1
Student copula (df=5, $\rho = 0.253$, $\lambda_L = \lambda_U = 0.107$)	1395.9	84.7
Gumbel copula ($\theta = 1.186$, $\lambda_L = 0$, $\lambda_U = 0.206$)	1445.1	87.7
Frank copula ($\theta = 1.631$, $\lambda_L = \lambda_U = 0$)	1279.1	77.6
Clayton copula ($\theta = 0.370$, $\lambda_L = 0.154$, $\lambda_U = 0$)	1234.4	74.9
Galambos copula ($\theta = 0.426$, $\lambda_L = 0$, $\lambda_U = 0.196$)	1450.6	88.0
Independence structure	1194.8	72.5
Comonotonic structure	1648.5	100.0
Unknown dependence structure	(274.7, 1791.5)	(16.6, 108.7)

Table 1. Research results.

Conclusions

The results of QIS5 presented in (*EIOPA Report...*,2017) show that the diversification effect may have a considerable impact on the decrease of solvency capital requirement of an insurer. The solution proposed as part of Solvency II, where the requirement is taken into account when determining SCR, on the one hand, belongs to the elements of awarding good risk management systems but, on the other hand, it requires that managers develop the right risk aggregation methods.

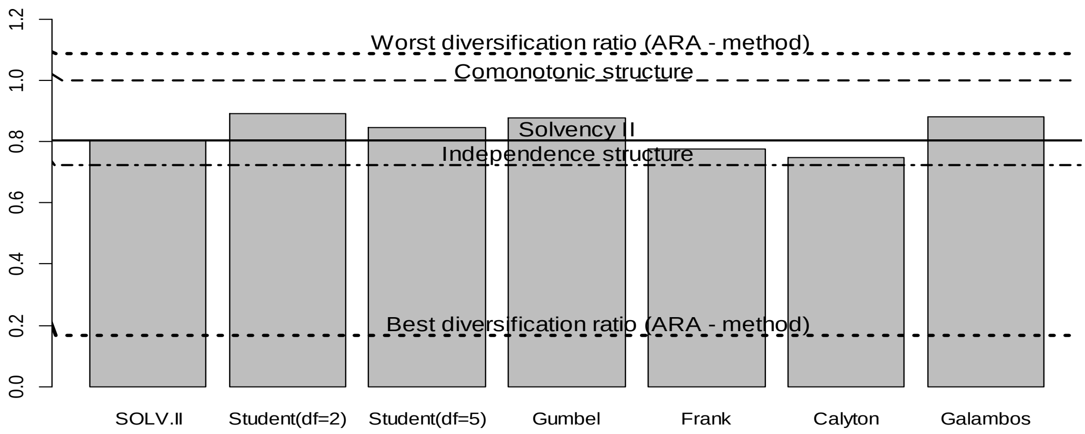

Fig. 1. Diversification ratio for the analysed dependence structures.

It should be emphasised here that the diversification effect is closely related to (or results from) the dependence structure between risks for which capital requirements are aggregated. Therefore, in order to estimate correctly the diversification effect, the structure must be identified properly. If only linear correlation coefficients are used for this purpose, this may cause errors in results as they unequivocally describe linear dependences only. In general, dependences between risks may be so complex that several numbers in the correlation matrix may not suffice for describing them (they may be non-linear, characterised by stronger dependences in tails, etc.). Furthermore, due to insufficient reliable data, correlation coefficients used in standard formulas depend to a considerable extent on the individual opinions of experts. Therefore, there is a need to carry out research that will focus on seeking new, more precise methods of recognising and modelling dependence structures as well as ways of including them in solvency models. In internal models (full or partial) or own parameters, Solvency II Directive allows or even encourages such research and implementation of nonstandard solutions into solvency models. New methods must be accepted by the market regulator.

Acknowledgements

The study is supported with subsidies for maintaining research capacity granted to the Faculty of Finance and Law at the Cracow University of Economics by the Polish Ministry of Science and Higher Education.

References

Bernard, C., Denuit, M., & Vanduffel, S. (2016). Measuring Portfolio Risk Under Partial Dependence Information. *Journal of Risk and Insurance.*

Bernard, C., Rüschendorf, L., & Vanduffel, S. (2015). Value-at-Risk Bounds With Variance Constraints. *Journal of Risk and Insurance*.

Dhaene, J., Goovaerts, M., Lundin, M., & Vanduffel S. (2005), *Aggregating economic capital*, Belgian Actuarial Bulletin, 5(1), 14-25.

EIOPA Report on the fifth Quantitative Impact Study (QIS5 ... (n.d.). Retrieved February 24, 2017 , from https://eiopa.europa.eu/Publications/Reports/QIS5_Report_Final.pdf.

Embrechts, P., Puccetti, G., & Rüschendorf, L. (2013). Model uncertainty and VaR aggregation. Journal of Banking & Finance, 37(8), 2750-2764.

Embrechts, P., Wang, B., & Wang, R. (2015). Aggregation-robustness and model uncertainty of regulatory risk measures. *Finance and Stochastics*, 19(4), 763-790.

Hofert, M., Memartoluie, A., Saunders, D., & Wirjanto, T. (2017). Improved algorithms for computing worst Value-at-Risk. *Statistics & Risk Modeling*, 0(0).

Lelyveld, I. V. (2006). *Economic capital modelling: concepts, measurement and implementation*. London: Risk Books.

Makarov, G.D. (1981). Estimates for the distribution function of the sum of two random variables when the marginal distributions are fixed. *Theory of Probability and its Applications*, (26), 803–806.

Puccetti, G., & Rüschendorf, L. (2012a). Bounds for joint portfolios of dependent risks. *Statistics & Risk Modeling*, 29(2), 107-132.

Puccetti, G., & Rüschendorf, L. (2012b). Computation of sharp bounds on the distribution of a function of dependent risks. *Journal of Computational and Applied Mathematics*, 236(7), 1833-1840.

Puccetti, G., & Rüschendorf, L. (2014). Asymptotic equivalence of conservative value-at-risk- and expected shortfall-based capital charges. *The Journal of Risk*, 16(3), 3-22.

Puccetti, G., Wang, B., & Wang, R. (2013). Complete mixability and asymptotic equivalence of worst-possible VaR and ES estimates. Insurance: Mathematics and Economics, 53(3), 821-828.

Rüschendorf, L. (1982). Random variables with maximum sums. *Advances in Applied Probability*, (14), 623–632.

QIS5 Technical Specifications - European Commission. (n.d.). Retrieved February 24, 2017, from https://eiopa.europa.eu/publications/qis/insurance

Wanat S. (2014). Ocena efektu dywersyfikacji ryzyka w Solvency II - aspekt praktyczny i metodologiczny. *Wiadomości Ubezpieczeniowe*, (3), 31-48.