On the impact of intraday trading volume on return’s volatility – a case of the Warsaw Stock Exchange

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Abstract
The aim of this paper is to present results of preliminary research in which the influence of intraday trading volume on return’s volatility on the Warsaw Stock Exchange is empirically examined. This study investigates whether the effect of intraday trading volume on return’s volatility is homogenous by dividing trading volume into its expected and unexpected components. We use 10-minute intraday data and measure return’s volatility by the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) structure using expected and unexpected components of trading volume as explanatory variables. We found that both higher expected and unexpected trading volumes are connected with a higher conditional return’s volatility. We also observed that unexpected volume shocks have a significantly larger effect on return’s volatility than changes in expected volume. Moreover, when volume is split into its expected and unexpected components and then incorporated into the conditional variance specification, GARCH effects are definitely reduced.

Keywords: volume-volatility relationship, trading volume, return volatility, ACV model, EGARCH

JEL Classification: C58, C11, C22

1. Introduction
The information role of trading volume in explaining price volatility and returns has generated a lot of interest for a long time. Hence, the dynamic relation between asset returns, return’s volatility and trading volume has been a subject of a considerable amount of research. It must be stressed that understanding of the volume-volatility relation might eventually lead to better volatility forecasting and a new and better way for modelling returns distributions. Moreover, analysis of the role of trading volume in explaining return’s volatility is of importance to policy makers and investors to better understand the response of the market to information shocks and dissemination of new information among market participants.

There are at least two theories in market microstructure literature that explain the volume-volatility relationship. The first one refers to the mixture of distributions hypothesis (MDH) developed by Clark (1973) and Epps and Epps (1976). According to this theory, a joint distribution of volume and volatility is conditional upon the flow of information into the market. Thus, both trading volume and volatility react and change contemporaneously in response to the arrival of new information.

On the other hand, the sequential information arrival hypothesis (SIAH) was advanced by Copeland (1976) and Jennings et al. (1981). This theory assumes that new market information is disseminated sequentially to traders. Therefore, the process in which new information is impounded into the price can spread out over time. It explains the lead-lag relation between volatility and trading volume.

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Beyond theoretical considerations, there is an extensive literature on the empirical aspects of the volatility-volume and return-volume relations. Empirical evidence on the volatility-volume relationship was found in numerous papers. The main results of theoretical frameworks were confirmed primarily by early studies of Clark (1973), Epps and Epps (1976), Jennings et al. (1981) and Karpoff (1987). However, the volatility-volume relation was also found by many other studies, among which are the following: Lamoureux and Lastrapes (1990), Gallant et al. (1992), Bessembinder and Seguin (1993), Jones et al. (1994), Gallo and Pacini (2000), Chen et al. (2001), Girard and Biswas (2007), Chevallier and Sevi (2012), Slim and Dahmene (2016), Ftiti et al. (2017) among others.

Many research papers usually document the positive correlation between trading volume and price volatility. However, in many cases there are not identical conclusions of the empirical research. Results range from lack of, to weak and strong relationships between return’s volatility and volume. Research even points out the possibility of a negative relation between volatility and trading volume. Furthermore, studies on this topic were conducted mostly for daily, weekly or monthly data. The application of intraday data is rather limited (Chevallier and Sevi, 2012; Slim and Dahmene, 2016; Ftiti et al., 2017). Moreover, the majority of empirical studies concentrate on well developed markets, especially on the U.S. market. Research on the volume-volatility relation for developing countries of Middle and Eastern Europe, such as Poland is still relatively sparse. The papers on the Polish financial market include Bohl and Henke (2003), Gurgul et al. (2005), Doman (2011), Bień-Barkowska (2012). Therefore, further insight should be obtained through different econometric methods as well as for the high frequency data and less developed markets, including Poland.

This study provides additional empirical evidence on the relations between price volatility and trading activity measured by the trading volume. The aim of this research is to present results of a pilot study in which the influence of intraday trading volume on return’s volatility on the Warsaw Stock Exchange is empirically examined. Following Bessembinder and Seguin (1993), this study investigates whether the effect of intraday trading volume on return’s volatility is homogenous by dividing trading volume into its expected (anticipated) and unexpected (unanticipated) components and allowing each component to have a separable effect on return volatility. In this study in order to define expected and unexpected trading volume variables, the autoregressive conditional volume (ACV) model of Manganelli (2005) is applied to describe the observed trading volume. To model return’s volatility, the exponential generalised autoregressive conditional heteroscedasticity (EGARCH) structure is employed using expected and unexpected components of trading volume as explanatory variables. An empirical study of the intraday volume-volatility relationship is performed for 10-minute intraday volume and return data of the main index of the Warsaw Stock Exchange. To estimate considered model, Bayesian approach is adopted. The Markov chain Monte Carlo (MCMC) methods including Metropolis-Hastings (MH) algorithm are suitably used to obtain posterior densities of interest.
2. Research methodology

In order to examine the influence of trading volume on return’s volatility, we must first distinguish between the expected and unexpected trading volume. It was well documented that the trading volume is highly autocorrelated, indicating that it is also highly forecastable. Thus, the expected volume is the result of more persistent fluctuations in liquidity whereas unexpected volume should approximate a new information arrival to the market. Trading volume decomposition into its expected and unexpected components is typically performed by means of ARMA models (Bessenbinder and Seguin, 1993; Bjonnes et al., 2003; among others). In this study in order to define expected and unexpected trading volume variables, the ACV model of Manganelli (2005) is applied to describe the observed trading volume. In particular we use the linear ACV(1,1) model with the generalised gamma distribution for the error term. This model for the volume \( v_i, i = 0, 1, 2, \ldots, N \) (with \( N \) standing for the total number of observations) can be written as follows:

\[
v_i = \Phi_i \cdot e_i, \tag{1}\n\]

\[
\Phi_i = E(v_i | \mathcal{I}_{i-1}, \theta), \tag{2}\n\]

where \( \mathcal{I}_{i-1} \) denotes the set of information available at time \( t_{i-1} \), \( \theta \) is the vector of unknown parameters, \( \Phi_i \) represents the conditional expected trading volume, \( e_i \) denotes an error term and \( \{e_i\} \sim i. i. d. GG (\lambda, \gamma, \nu) \) with parameter \( \lambda = \left( \frac{\Gamma \left( \frac{\nu}{\gamma} \right)}{\Gamma \left( \frac{1}{\gamma} \right)} \right) \) such that expected value \( E(e_i) = 1 \).

The conditional expectation of trading volume has the following representation:

\[
\Phi_i = \omega + \alpha \cdot v_{i-1} + \beta \cdot \Phi_{i-1}, \tag{3}\n\]

where \( \omega > 0, \alpha \geq 0, \beta \geq 0, \alpha + \beta < 1 \). Accordingly, the expected volume \( v_{\text{exp},i} \) is defined as an estimate of the conditional expectation of volume \( \hat{\Phi}_i \), whereas the unexpected volume \( v_{\text{unexp},i} \) is defined as ratio of observed volume to expected volume \( v_{\text{unexp},i} = \frac{v_i}{\hat{\Phi}_i} \).

Now we proceed to dynamic specification for the 10-minute financial logarithmic returns. We assume a simple AR (1) structure for the intraday returns:

\[
r_i - \delta = \rho \cdot (r_{i-1} - \delta) + u_i, \tag{4}\n\]

\[
u_i = \sigma_i \cdot \xi_i, \tag{5}\n\]

where \( -1 < \rho < 1, \xi_i \) is the innovation term, \( \{\xi_i\} \sim i. i. d. t (0; 1; \nu), \nu > 2 \) and \( \sigma_i^2 \) is the conditional variance of the returns. By \( t (0; 1; \nu) \) we denote Student’s \( t \) distribution with zero mean, unit precision and an unknown number of degrees of freedom \( \nu > 2 \). To model return’s volatility, we specify the EGARCH structure with expected and unexpected components of trading volume as explanatory variables. In fact, we propose an EGARCH (1,1)-type specification of the conditional variance, the dynamics of which evolves according to the following equation:

\[
\ln \sigma_i^2 = \omega_G + \alpha_{1G} \cdot \xi_{i-1} + \alpha_{2G} \cdot \left( \left| \xi_{i-1} \right| - \frac{2 \Gamma \left( \frac{2}{\nu} \right) \sqrt{\nu - 2}}{\Gamma \left( \frac{\nu}{2} \right) \Gamma \left( \frac{\nu}{2} \right)} \right) + \beta_G \cdot \ln \sigma_{i-1}^2 + \eta_1 \cdot v_{\text{exp},i} + \eta_2 \cdot v_{\text{unexp},i}. \tag{6}\n\]
This specification allows for an asymmetric response of $\sigma_i^2$ to volatility shocks in the innovation term $\xi_{i-1}$ when parameter $\alpha_{iG}$ differs from zero. The use of an EGARCH-type model is also justified by the advantage of keeping the volatility component positive regardless of the sign of the right-hand side components in the volatility equation. The absence of non-negativity constraints on the parameters also facilitates numerical estimation.

3. Bayesian estimation of the AR-EGARCH-X and ACV models

In order to estimate parameters of the proposed models, the Bayesian approach is applied. Bayesian estimation of the AR-EGARCH-X and ACV models outlined above requires certain prior assumptions. We assume that all parameters – whenever possible – are a priori independent. Moreover, in order to express the lack of prior knowledge, fairly diffuse prior distributions are assumed, so that the data dominates the inference about the parameters through the likelihood function. Specifically, for all parameters of Equation (3) we propose the normal distributions with zero mean and standard deviation of five, adequately truncated, due to relevant restrictions imposed on the parameters in the model. For the ACV model with the generalised gamma innovations, the prior density for parameter $\gamma$ is also specified as density of the normal distribution with zero mean and standard deviation of five adequately truncated whereas the prior density for parameter $\nu$ is specified as density of the normal distribution with zero mean and standard deviation of ten adequately truncated. The prior for the parameter $\rho$ is set to be uniform over the $(-1;1)$ interval. Prior for degrees of freedom $\nu$ is practically uniform and assumes that the parameter is restricted to the range $(2; 102)$ since it ensures that the conditional variance exists. The upper bound is used for numerical convenience only and the restriction has very limited empirical consequences. For remaining parameters of Equations (4) and (6) we propose the normal distributions with zero mean and standard deviation of five, adequately truncated if necessary.

The inference was conducted using MCMC techniques. The MH algorithm with a multivariate Student’s $t$ candidate generating distribution with three degrees of freedom and the expected value equal to the previous state of the Markov chain was used to generate a pseudorandom sample from the posterior distribution. The covariance matrix was obtained from initial cycles, which were performed to calibrate the sampling mechanism. Convergence of chain was carefully examined by starting the MCMC scheme from different initial points and checking trace plots of iterates for convergence to the same posterior. Acceptance rates were sufficiently high and always exceeded 50%, indicating good mixing properties of the posterior sampler. The final results and conclusions were based on 100,000 draws, preceded by 50,000 burn-in cycles. All codes were implemented by the author and ran using the GAUSS software, version 13.0.

4. Data description

The empirical analysis is based on 10-minute intraday data of the WIG20 Index of the Warsaw Stock Exchange (WSE). Our sample consists of transaction prices and trading volumes matched
for each time interval. The analysis covers the period from January 12th, 2018 to April 11th, 2018 (62 trading days), and is based on transactions carried out in the continuous trading phase which in the case of the WSE in 2018 starts at 09:00 and ends at 17:20 (GMT+1). The data are obtained from the Thomson Reuters Eikon Database. From the data, we generate the 10-minute index return series by taking the log of the ratio of last transaction prices in successive intervals.

It is well documented in the financial literature the existence of intraday periodicities in intraday returns and trading volumes. The intraday periodic patterns for intraday returns and volumes were estimated using the Nadaraya-Watson kernel estimator of regression of the variable on the time of the day. Finally, intraday returns and trading volumes were deseasonalised by dividing plain data by diurnal factor to obtain diurnally adjusted data.

5. Empirical results
Bayesian estimation results of the proposed AR-EGARCH-X model, including marginal posterior means and standard deviations (in parentheses), are reported in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pure AR-EGARCH model</th>
<th>AR-EGARCH-X model with expected and unexpected volume variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>$-0.0311 (0.0197)$</td>
<td>$-0.0206 (0.0185)$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$-0.0507 (0.0155)$</td>
<td>$-0.0688 (0.0161)$</td>
</tr>
</tbody>
</table>

The volatility equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pure AR-EGARCH model</th>
<th>AR-EGARCH-X model with expected and unexpected volume variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_G$</td>
<td>$0.0286 (0.0156)$</td>
<td>$-1.3159 (0.1424)$</td>
</tr>
<tr>
<td>$\alpha_{1G}$</td>
<td>$-0.0383 (0.0120)$</td>
<td>$-0.0649 (0.0329)$</td>
</tr>
<tr>
<td>$\alpha_{2G}$</td>
<td>$0.0971 (0.0233)$</td>
<td>$0.1783 (0.0498)$</td>
</tr>
<tr>
<td>$\beta_G$</td>
<td>$0.9658 (0.0176)$</td>
<td>$0.1714 (0.0780)$</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>$\cdots$</td>
<td>$0.7417 (0.1349)$</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>$\cdots$</td>
<td>$1.0537 (0.0723)$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$3.2935 (0.1761)$</td>
<td>$4.5526 (0.3995)$</td>
</tr>
</tbody>
</table>

Looking at the results for the return equation parameters, it can be noted that the posterior mean of the autoregressive coefficient ($\rho$) is negative and equal to $-0.0688$. Moreover, the posterior distribution of $\rho$ is well-separated from zero and features relatively little dispersion indicated by the standard deviation of about 0.0161. Moreover, the conditional normality of the intraday returns is strongly overridden by the data. The posterior mean and standard deviation of the degrees of freedom equal about 4.5526 and 0.3995, respectively. Therefore, our results
confirm that allowing for fat tails of the conditional distribution may be crucial for empirically adequate statistical modelling with the use of GARCH-type processes.

As far as the variance equation is concerned, it can be seen that the posterior results for parameter $\beta_G$ imply fairly weak volatility persistence. The posterior mean of $\beta_G$ stands at about 0.1714. The asymmetry effect is slightly negative, but it seems to be statistically insignificant. The posterior mean of $\alpha_1$ is equal to $-0.0649$ and is accompanied by a relatively large posterior standard deviation of about 0.0329.

Finally, we analyse the estimation results for parameters $\eta_1$ and $\eta_2$ pertaining to the effects of trading volume on conditional variance. The posterior distributions of both parameters are well-separated from zero, indicating statistical significance of trading volume variables. The posterior mean of $\eta_1$ (related to the expected trading volume, $v_{exp,i}$) is positive and equal to 0.7417, suggesting that positive impact of the expected volume on return’s volatility. Thus, higher expected trading volume is connected with a higher volatility. The posterior mean of $\eta_2$ (related to the unexpected trading volume, $v_{unexp,i}$) is also positive, amounting to about 1.0537. The posterior distribution of the parameter under consideration reveals rather a relatively small dispersion which is implied by the standard deviation of about 0.0723. It follows that higher unexpected trading volume is connected with a higher volatility. It can also be noted that unexpected shocks have larger effect on return’s volatility than changes in expected volume. Thus, surprises in trading activity measured by trading volume have a larger effect on return’s volatility than forecastable trading activity. Moreover, when volume is split into its expected and unexpected components and then incorporated into the conditional variance specification, GARCH effects are definitely reduced compared to pure EGARCH specification (see Table 1).

Conclusions
The main objective of this paper is to present results of a pilot study in which the influence of intraday trading volume on return’s volatility on the WSE is empirically examined. We used 10-minute intraday data and measured return’s volatility by the exponential generalized autoregressive conditional heteroscedasticity structure using expected and unexpected components of trading volume as explanatory variables.

Our main findings can be summarised as follows. The results suggest that trading volume has a significant impact on the return volatility of the main index of the WSE. We found that both higher expected and unexpected trading volume are connected with a higher conditional return’s volatility. We also showed that unexpected volume shocks have significantly stronger effect on return’s volatility than changes in expected volume. Moreover, when volume is divided into its expected and unexpected components and then incorporated into the conditional variance specification, GARCH effects are definitely reduced. These findings demonstrate the importance of splitting the total trading volume into expected and unexpected components. Finally, it must be stressed that our analysis was limited only to the main index of the WSE. So it would be of great interest to examine more indices from other well-developed and emerging
markets in further research. Moreover, the issue that needs further examination is to analyse particular stocks included in the WIG20 index, which display substantial cross-sectional differences resulting from different capitalization. It will allow us to better understand the differences across various stocks and market structures.

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References


